Kyushu Algebraic Number Theory 2017 —Abstract—

March 8, 2017 (Wed)

- **Shushi HARASHITA (Yokohama National University)**
  Superspecial curves of genus 4 in small characteristic
  A superspecial curve is a (non-singular projective) curve whose Jacobian is the product of supersingular elliptic curves. It is known that there are only finitely many isomorphism classes of superspecial curves of a given genus in a given characteristic. We are interested in enumerating them. In the case of polarized abelian varieties, some theories and some general results are known, but in the case of curves, there are still many open problems. After recalling some results in the case of abelian varieties and some theoretical backgrounds for curves, I am going to talk about the enumeration of superspecial curves of genus 4 in small characteristic, which was obtained in a joint work with Momonari Kudo.

- **Yoshinori MISHIBA (National Institute of Technology, Oyama College)**
  Logarithmic Interpretation of Carlitz Multizeta Values
  Carlitz multizeta values (CMZVs) were defined by Thakur as function field analogues of multiple zeta values in characteristic 0. Anderson and Thakur showed that Carlitz zeta values (depth one CMZVs) appear in a coordinate of the logarithmic function of a tensor power of the Carlitz t-module. In this talk, we generalize this for higher depth cases. Thus, we construct a t-module for each higher depth CMZV. Then we prove that each CMZV appears in a coordinate of the logarithmic function of this t-module. This is a joint work with Chieh-Yu Chang.

- **Yoshiyasu OZEKI (Kanagawa University)**
  Lattices in crystalline representations and Kisin modules associated with iterate extensions
  Cais and Liu extended the theory of Kisin modules and crystalline representations to allow more general coefficient fields and lifts of Frobenius. Based on their theory, we classify lattices in crystalline representations by Kisin modules with additional structures under a Cais-Liu's setting. Furthermore, we give a full faithfulness theorem for torsion crystalline representations.

March 9, 2017 (Thu)

- **Yoshiaki OKUMURA (Tokyo Institute of Technology)**
  A function field analogue of the Rasmussen-Tamagawa conjecture
  Let K be a finite extension of $\mathbb{F}_q(t)$ and $\mathbb{G}_a$ the additive group over K. A Drinfeld module over K, which is an analogue of elliptic curves over number fields, is an $\mathbb{F}_q$-algebra homomorphism $\varphi : \mathbb{F}_q[t] \to \mathrm{End}_{\mathbb{F}_q}(\mathbb{G}_a)$ with some conditions. In this talk, for Drinfeld modules over K, we will discuss an analogue of a non-existence conjecture for abelian varieties over number fields suggested by Christopher Rasmussen and Akio Tamagawa, and explain some results about it.

- **Koji TASAKA (Aichi Prefectural University)**
  Period polynomial relation between double zeta values revisited
  Double zeta values and elliptic cusp forms are intimately related each other, which was first discovered by Zagier for depth 2. In 2006, Gangl, Kaneko and Zagier constructed an explicit correspondence from elliptic cusp forms, or rather, their restricted even period polynomials to linear relations among certain double zeta values modulo a Riemann zeta value. This linear relation among double zeta values is called period polynomial relations. Although one can compute the coefficient of the Riemann zeta value in the period polynomial relation, it is not quite explicit. In this talk, a more elaborate correspondence from even period polynomials corresponding to cusp forms to linear relations among double zeta values and a single zeta value is given. The proof uses motivic 1/2-double zeta values and a rational associator of depth 2 constructed by Brown.

- **Tomoaki NAKAYA (Kyushu University)**
  On modular solutions of certain modular linear differential equation and supersingular polynomials
  The supersingular polynomial is a polynomial over $\mathbb{F}_p$ whose roots are exactly supersingular j-invariants in characteristic p. It is known that the supersingular polynomial can be expressed by using a hypergeometric polynomial, and there is a way to construct this polynomial from a modular solution of a certain differential equation. In this talk, we generalize this construction to a differential equation with several parameters.

- **Masataka ONO (Keio University)**
  Finite multiple zeta values associated with 2-colored rooted trees
  Finite multiple zeta values (FMZVs) first defined by Kaneko and Zagier studied by many mathematicians. Kamano introduced other type FMZVs called Mordell-Tornheim type (MT-type) and obtained an explicit formula of FMZVs of MT-type in terms of FMZVs. As a corollary, Kamano obtained many linear relations among FMZVs. In this talk, we introduce 2-colored rooted trees, which are some combinatorial objects, and define FMZVs associated with 2-colored rooted trees. We will show that they can be regarded as common generalizations of FMZVs and FMZVs of MT-type. Moreover, we will explain that with a mild assumption, FMZVs associated with 2-colored rooted trees can be written as a sum of FMZVs and give another proof of the shuffle relation among FMZVs as a corollary.

- **Shin-ichiro SEKI (Osaka University)**
  The $\boldsymbol{p}$-adic number ring and $\widehat{\mathcal{A}}$-finite multiple zeta values
  We will introduce the $\boldsymbol{p}$-adic number ring $\widehat{\mathcal{A}}$ and define the $\widehat{\mathcal{A}}$-finite multiple zeta value ($\widehat{\mathcal{A}}$-FMZV) as an element of $\widehat{\mathcal{A}}$. We will explain previous works by Rosen and results obtained by the speaker for relations among $\widehat{\mathcal{A}}$-FMZVs.

March 10, 2017 (Fri)

- **Nao TAKESHI (Gakushuuin University)**
  Determining all elliptic curves with good reduction everywhere over number fields
  It is known that if K is a number field and S is a finite set of primes of K, there are only finitely many elliptic curves defined over K with good reduction outside S up to K-isomorphism. I will describe an algorithm to determine the finite set of such elliptic curves over a given K with $S = \emptyset$. Examples are given over number fields of small degree. The strategy of our algorithm is to give a finite set of possible values for the j-invariants, which is constructed of solutions of unit equations. I will also describe a family of elliptic curves having good reduction everywhere over number fields of the same degree as the j-invariants, constructed of solutions of certain unit equations.

- **Kazuhiro YOKOYAMA (Rikkyo University)**
  On Symbolic Formula of Isogeny of Elliptic Curves
  We argue a symbolic formula of isogeny of elliptic curves which is expressed in terms of parameters of elliptic curves. First we show that such a symbolic formula exists and characterize its form by theoretical manner. Then, we show that such a formula can be computed by solving certain system of algebraic equations derived directly from the well-known Vélu’ s formula. As finding such a formula is reduced to computation of a Gröbner basis of the ideal corresponding to the system, we report how our techniques for efficient Gröbner basis computation work on this computation.

- **Tetsuya TANIGUCHI (Kanazawa Institute of Technology)**
  On the specificity of the magnitude of the determinant formula of the relative class number of cyclotomic fields and its application
  In this presentation, we report some specificity of the magnitude of the values of the determinants of the relative class numbers of cyclotomic fields. The Demjanenko matrix is a matrix of 0, 1 components, and the value of its determinant is known to represent the relative class number of a cyclotomic field. We have obtained the observation that its absolute value is extremely larger than the absolute value of the determinant of a random 0-1 matrix.
  We also observe similar peculiar phenomena in the determinants of relative class numbers with $\pm$ 1 components. We expect that the determinant formulas for relative class numbers can be applied to the construction of matrices with large determinant values.
  In this talk, we introduce the determinant formulas for several kinds of relative class numbers and compare their magnitude with that of the determinants of matrices with random coefficients. We also mention that we are considering the design of experiments, etc. as an application of this peculiar property of the determinant formulas for relative class numbers. In addition, we will also describe an attempt to apply the fast computation method of the relative class numbers of cyclotomic fields we had done before to the calculation of these determinants.

- **Hiroki TAKAHASHI (Tokushima University)**
  Generalized Greenberg's conjecture for cyclotomic fields and special elements of K-groups
  Let p be a prime number, k a finite extension of $\mathbf{Q}$ and $\tilde{k}$ the compositum of all $\mathbf{Z}_p$-extensions of k. Generalized Greenberg's conjecture states that the Galois group of the maximal unramified abelian pro-p extension of $\tilde{k}$ is pseudu-null as a $\mathbf{Z}_p[[\mathrm{Gal}(\tilde{k}/k)]]$-module. McCallum and Sharifi gave sufficient conditions for the conjecture for $k = \mathbf{Q}(\zeta_p)$ using special elements of K-groups which are constructed from cyclotomic units, and checked the conjecture for small prime numbers p. In this talk, I will report some computational results on a similar investigation for $k = \mathbf{Q}(\zeta_{4p})$.